An Analytical Investigation of Shape Control of Large Space Structures by Applied Temperatures

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An analytical procedure for the static shape control of flexible space structures is described. The procedure is based on prescribing temperatures in control elements which have much higher coefficients of thermal expansion than the main structure. The temperatures at the control elements are determined so as to minimize the overall distortion of the structure from its ideal shape. A matrix equation is derived and solved for the optimum control temperatures. The paper contains two formulations of the procedure: one for continuous structures governed by differential equations, and the other for discrete (finite element modeled) structures. The equations from the continuous formulation have been applied to shape control of a simple beam distorted by nonuniform heating. Use of only seven control elements was able to achieve a factor of 50 reduction in thermal distortion. The discrete formulation has been implemented in a general-purpose finite element structural analysis computer program and demonstrated for shape control of a 55-m radiometer antenna reflector subjected to orbital heating. Results for the antenna were obtained for four different sets of control locations. Reductions in distortion of up to factor of three were realized.

Nomenclature

A = matrix in control equation, Eq. (12) or (48)
B = matrix defining distortion measure, Eq. (45)
C = matrix defined by Eq. (59)
E = Young's modulus
E_c = Young's modulus of control elements
E_0 = Young's modulus of the beam
F = expanded vector of control forces, Eq. (52)
F_i = magnitude of ith control force
G = rms reduction factor, Eq. (14)
I = moment of inertia of the beam
L = length of the beam
L_i = location vector for ith control force, Eq. (52)
M = mass matrix
M_{Ty} = thermal bending moment in the beam
m = total mass of structure
n = number of control points
r = right-hand side of control equation, defined by Eq. (13) or (49)
R_i = ith rigid-body mode
S = cross-sectional area of the beam
S_0 = cross-sectional area of one flange of the beam
T = temperature
u = displacement field
u_0 = displacement field of supported structure
u_i = displacement field of free structure due to unit T_i or F_i
u_{0i} = displacement field of supported structures due to unit T_i or F_i
U_i = displacement vector of free structure due to unit T_i or F_i
U_{0i} = displacement vector of supported structure due to unit T_i or F_i
U = displacement vector
u_T = corrected displacement field
U_T = corrected displacement vector
U_{zi} = z component of displacement at ith node
v_0 = reference volume
w = beam transverse displacement
z_0 = beam coordinate (see Fig. 1)
α = coefficients of rigid-body motion in displacement field
α_0 = thermal expansion coefficient
α_c = thermal expansion coefficient of control element
β = coefficients of rigid-body motion of distortion vector
γ_i = defined by Eq. (27)
Γ = defined by Eq. (55)
ΔL_i = length of ith control element in the beam
ΔS_i = cross-sectional area of ith control element in the beam
ΔT = vector of control temperatures
ΔT_i = control temperature at ith control point
ΔT_0 = disturbance temperature in the beam
ρ = mass density
ϕ = augmented objective function, Eq. (17)
ψ = distortion vector
Ω = domain of definition of structure

Introduction

In the design of large space antennas, one of the more stringent design requirements is that of surface accuracy.[1,2] While studies have shown that in some cases high-surface accuracies may be maintained with passive methods,[3] it is expected that for many applications active controls may be needed. The disturbances which affect the shape of space structures may be divided into two types. One type is transient, which leaves the structure unchanged once damped out. Such disturbances usually call for active or passive controls which enhance the damping of the structure. The second type of disturbance is typified by fixed deformations (e.g., due to manufacturing errors[4]) or those which are slowly varying and may be considered quasisteady. These latter disturbances may be offset by slowly applied, long acting corrections. Most research to date has concentrated on the first type of

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disturbance and the use of damping actuators.[5] There has been less research on countering quasisteady disturbances. Most of the work on active control of quasisteady disturbances is related to active control of optical systems, such as mirrors (see Ref. 6 for a survey of the state-of-the-art in 1978). Most of the actuators employed are force actuators.[7-11] Bushnell[7] characterizes some such actuators[12,13] as displacement actuators because they are stiff enough to enforce a given displacement at a point. Another variation of the force actuator is one which effects a change in the length of a member by reeling a cable in or out or by a screw mechanism in a truss structure. This type of approach is used on some antennas[14] to correct fabrication errors, albeit on the ground rather than in orbit.

A major source of quasisteady disturbances of orbiting space structures is thermal loading. To minimize thermal deformations, it is expected that large space structures will utilize composite materials, which may be tailored to have small coefficients of thermal expansion. In this type of design, the heating or cooling of small elements made of materials having large coefficients of thermal expansion (e.g., aluminum) may be an attractive form of control. Applied heating control has the following attractive features:

1) Thermal loads are self equilibrating; therefore, one need not worry about small inaccuracies in equilibrating control forces which lead to drift in position or orientation.

2) Solar energy is available for heating.

3) For a free body in space, stresses associated with applied heating are smaller than those associated with applied forces.

The present paper addresses the use of applied heating and cooling of actuator elements in the structures to control the quasisteady types of deformations. For truss elements, applied temperature control is similar in its effect to the control achieved by adjusting the length of an element. However, the implementation is quite different and is equally applicable to one-, two-, and three-dimensional elements in a structure. Subsequent sections of this paper describe the development of a set of equations to obtain the required temperatures to control a given disturbance. The equations are first derived for a continuous structure and then adapted to a discrete, finite element modeled structure. The finite element formulation is implemented in the EISI/EAL finite element structural analysis system.[15] Two example problems are employed for demonstration. The first is a free-free beam where the continuous formulation is employed, and the second is a finite element model of the reflector dish of a 55-m radiometer antenna. For each problem, both thermal and force controls are used and their effectiveness is compared.

Equations for Shape Control of Stuctures—Continuous Formulation

Control by Applied Temperatures

The structure is assumed to be in Earth orbit and unrestrained against rigid-body motion. The structure is defined over some region Ω and it is assumed that its desired shape has been distorted by an amount which is described by a displacement vector $\psi(Q)$‡ where Q is a point in Ω. It is also assumed that the center of gravity (c.g.) of the body does not rotate or translate in space; that is,

$$\int_\Omega \rho\psi\cdot\boldsymbol{R}_i \mathrm{d}\Omega = 0 \qquad i=1,2,...,6 \tag{1}$$

where R_i is the ith rigid-body mode, ρ the density of the stucture, and a dot denotes a scalar product. If Eq. (1) is violated, the correction is a matter for position and orientation control rather than shape control, which is outside the scope of this study.

The distortion of the structure is to be corrected by prescribing temperatures at selected locations. It is assumed that control elements with large coefficients of expansion relative to the rest of the structure are installed at these locations. Also, the control elements are assumed to be very small compared to the rest of the structure and can be considered to be discrete points.

The first step in the analysis is to solve for the deformation of the free structure in space subject to point heating. It is convenient to start by calculating the deformations when the structure has been arbitrarily supported against all rigid-body motion. The displacement field so obtained is denoted $\boldsymbol{u}_0$. Because this displacement field is based on an arbitrarily restrained structure, the c.g. of the body has not remained stationary. This violation is corrected by adding the contribution of rigid-body modes to give

$$\boldsymbol{u} = \boldsymbol{u}_0 + \sum_{i=1}^{6} \alpha_i \boldsymbol{R}_i \tag{2}$$

where the α_i are obtained from the condition

$$\int_\Omega \rho\boldsymbol{u}\cdot\boldsymbol{R}_i \mathrm{d}\Omega = 0 \qquad i=1,2,...,6 \tag{3}$$

The rigid-body modes are normalized so that

$$\int_\Omega \rho\boldsymbol{R}_i\cdot\boldsymbol{R}_j \mathrm{d}\Omega = m\delta_{ij} \tag{4}$$

where δ_{ij} is the Kronecker delta and m the mass of the structure. With this normalization, Eqs. (2) and (3) yield

$$\alpha_i = -\frac{1}{m}\int_\Omega \rho\boldsymbol{u}_0\cdot\boldsymbol{R}_i \mathrm{d}\Omega \tag{5}$$

The structure is controlled by applying temperatures ΔT_i, $i=1,...,n$ at n control points. Denote the displacement field corresponding to a unit temperature at the ith control point as $\boldsymbol{u}_i$. Then

$$\boldsymbol{u}_i = \boldsymbol{u}_{0i} + \sum_{k=1}^{6} \alpha_{ik} \boldsymbol{R}_k \tag{6}$$

where $\boldsymbol{u}_{0i}$ is the solution for a unit temperature at the ith control point with the structure supported against rigid-body motion and α_{ik} are obtained from $\boldsymbol{u}_{0i}$ via Eq. (5). That is,

$$\alpha_{ik} = -\frac{1}{m}\int_\Omega \rho\boldsymbol{u}_{0i}\cdot\boldsymbol{R}_k \mathrm{d}\Omega \tag{7}$$

The total displacement is the sum of the disturbance and the correction

$$\boldsymbol{u}_T = \psi + \sum_{i=1}^{n} \boldsymbol{u}_i \Delta T_i \tag{8}$$

The best values of ΔT_i are those which will most effectively nullify ψ, that is, cause $\boldsymbol{u}_T$ to be close to zero. A common measure of the smallness of $\boldsymbol{u}_T$ is the rms value

$$u_{\mathrm{rms}}^2 = \frac{1}{v_0}\int_\Omega \boldsymbol{u}_T\cdot\boldsymbol{u}_T \mathrm{d}\Omega \tag{9}$$

where v_0 is a reference volume.

‡In some cases part of the distortion does not need to be corrected; for example, for parabolic antenna surfaces, it is reasonable to measure the distortion from the best fit paraboloid rather than the original shape. It is assumed here that ψ is the part of the distortion that needs to be corrected.

The necessary condition for a minimum value of distortion is

$$\frac{\partial u_{rms}^2}{\partial \Delta T_j} = (2/v_0) \int_\Omega \left(\psi + \sum_{i=1}^{n} u_i \Delta T_i\right) \cdot u_j \mathrm{d}\Omega = 0 \qquad j=1,2,\ldots,n \tag{10}$$

Equation (10) is a system of n linear algebraic equations for the vector of control temperatures, ΔT which may be written as

$$A\Delta T = r \tag{11}$$

where the component a_{ij} of the matrix A is

$$a_{ij} = \frac{1}{v_0} \int_\Omega u_i \cdot u_j \mathrm{d}\Omega \tag{12}$$

and the jth component of the right-hand side, r is

$$r_j = -\frac{1}{v_0} \int_\Omega \psi \cdot u_j \mathrm{d}\Omega \tag{13}$$

The ratio of controlled to uncontrolled rms distortion, G is given by

$$G^2 = \int_\Omega u_T \cdot u_T \mathrm{d}\Omega \Big/ \int_\Omega \psi \cdot \psi d\Omega \tag{14}$$

It follows that

$$G^2 = \frac{r_{m0}^2 - 2r^T \Delta T + \Delta T^T A \Delta T}{r_{m0}^2} = 1 - \frac{r^T \Delta T}{r_{m0}^2} \tag{15}$$

where

$$r_{m0}^2 = \frac{1}{v_0} \int_\Omega \psi^2 \mathrm{d}\Omega$$

Control by Applied Forces

The analysis for control by applied forces is complicated by the requirement that the forces be self equilibrating. As before, it is assumed that there are n control points. The displacement shape u_{0i} due to a unit control force at the ith control point is obtained for a convenient set of boundary conditions that suppress rigid-body motion. Then the rigid-body mode contribution is added as in Eq. (6). The condition that the applied forces are in equilibrium is written as

$$\bar{R}_k^T F = 0 \qquad k=1,2,\ldots,6 \tag{16}$$

where F is the vector of control point forces and $\bar{R}_k$ a vector containing the portion of the kth rigid-body mode corresponding to n control points. Employing Lagrange multipliers, the augmented objective function is defined as

$$\phi = u_{\mathrm{rms}}^2 - \sum_{k=1}^{6} \lambda_k \bar{R}_k^T F \tag{17}$$

where, in this case,

$$u_{\mathrm{rms}}^2 = (1/v_0) \int_\Omega \left(\psi + \sum_{i=1}^{n} u_i F_i\right) \cdot \left(\psi + \sum_{i=1}^{n} u_i F_i\right) \mathrm{d}\Omega \tag{18}$$

minimize ϕ with respect to F_j and λ_k

$$\frac{\partial \phi}{\partial F_j} = 0 \quad j=1,2,\ldots,n \qquad \frac{\partial \phi}{\partial \lambda_k} = 0 \quad k=1,2,\ldots,6 \tag{19}$$

The resulting equation is

$$AF - r - \sum_{k=1}^{6} \lambda_k \bar{R}_k = 0 \tag{20}$$

where the components of A and r are given by Eqs. (12) and (13). Equations (20) and (16) constitute a system of $n+6$ equations with $n+6$ unknowns.

Application to a Free-Free Beam

Problem Description

The continuous formulation is applied to the control of thermal distortion of a free-free beam shown in Fig. 1. The beam cross section is composed of two flanges each of area S_0 separated by a thin web. The centroid of each flange is at a distance z_0 from the neutral axis of the beam. The beam is assumed to deform only in the x-z plane and the diplacement to be controlled is the z displacement of the neutral axis denoted w. The differential equation which describes thermal deformation of the beam is

$$\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} = -\frac{M_{Ty}}{EI} \tag{21}$$

where E is Young's modulus and I the moment of inertia of the beam cross section about the y axis. M_{Ty} is the thermal moment given by

$$M_{Ty} = \int_S \alpha_0 E \Delta T z \mathrm{d}S \tag{22}$$

where S is the cross-sectional area of the beam, and α_0 the coefficient of expansion. Consider the distortion ψ to be caused by a uniform heating ΔT_0 of the top left half of the beam. The thermal moment is given by

$$\begin{aligned} M_{Ty0} &= \alpha_0 E_0 z_0 S_0 \Delta T_0 && \text{for } x \le L/2 \\ &= 0 && \text{for } x > L/2 \end{aligned} \tag{23}$$

The distortion is

$$\begin{aligned} \psi &= \frac{\gamma_0 \Delta T_0}{L}\left(\frac{x^2}{2} - \frac{13}{32}xL + \frac{11}{192}L^2\right) && \text{for } x \le L/2 \\ &= \frac{\gamma_0 \Delta T_0}{L}\left(\frac{3Lx}{32} - \frac{13L^2}{192}\right) && \text{for } x > L/2 \end{aligned} \tag{24}$$

where $\psi_0 = \alpha_0 z_0 S_0 L/I$.

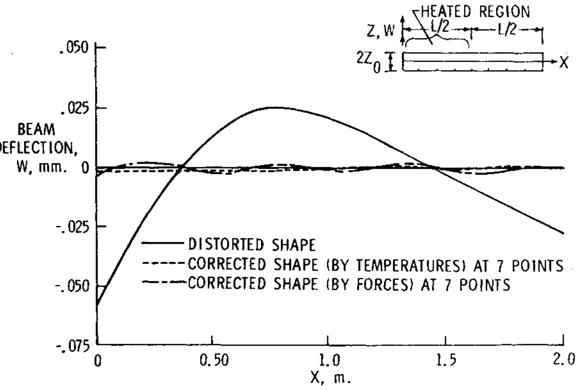


Fig. 1 Control of beam shape distortion.

Table 1 Effect of number of control points on thermal distortion correction for beam

No. of control points	Reduction in rms deformation,[a] G due to Applied temperatures	Applied forces
1	0.4220	—
2	0.1410	—
3	0.0963	0.3580
4	0.0517	0.0581
5	0.0411	0.0316
6	0.0268	0.0197
7	0.0219	0.0190

[a] $G = \dfrac{U_{\text{rms corrected}}}{U_{\text{rms uncorrected}}}$; $U_{\text{rms}} = \left[\sum_i U_{zi}^2\right]^{1/2}$.

Control by Applied Temperatures

It is assumed that the corrective heating is localized to a small volume of each control point. Assuming that control point i is at coordinates (x_i, z_i) and has an area ΔS_i and length ΔL_i, then approximately

$$M_{Ty_i} = \alpha_c E_c z_i \Delta S_i \Delta T_i \tag{25}$$

This thermal moment produces a slope change

$$\frac{dw}{dx}\bigg|_{x_i^+} = \frac{dw}{dx}\bigg|_{x_i^-} - \gamma_i \Delta T_i \tag{26}$$

where

$$\gamma_i = \alpha_c E_c z_i \Delta S_i \Delta L_i / EI \tag{27}$$

The next step is to find the displacement w_{T0i} due to a unit ΔT_i with boundary conditions which suppress rigid-body motion. It is convenient to choose zero displacement and slope at $x=0$. The solution for a unit ΔT_i is then

$$w_{Ti0} = 0 \qquad x \leq x_i$$

$$= -\gamma_i (x - x_i) \qquad x_i > x_i \tag{28}$$

The effect of the local change in density due to the control elements is neglected and the beam is assumed to have a uniform cross section. Under these assumptions the condition of rigid-body made orthogonality, Eq. (4), becomes

$$\frac{1}{L}\int_0^L R_i R_j \, dx = \delta_{ij} \tag{29}$$

The appropriate rigid-body modes are the plunge and pitch modes

$$R_1 = 1, \qquad R_2 = \sqrt{12}\,[(x/L) - 0.5]$$

In accordance with Eqs. (6) and (29), the complete expression for the unit response function is given by

$$w_{Ti} = w_{Ti0} + \alpha_1 + \sqrt{12}\alpha_2 (x/L - 0.5) \tag{30}$$

Details of the calculations of α_1, α_2, the A matrix [Eq. (12)], and the r vector [Eq. (13)] have been omitted from this paper but are included in Ref. 16. The control points are equally spaced along the beam. Since thermal controls are ineffective at the edges, the thermal control points are located at

$$x_i = iL/(n+1) \qquad i = 1,2,\ldots,n$$

The improvements in rms distortion as given by the parameter G [Eq. (14)] is given in Table 1 as a function of the number of control points.

Control by Applied Forces

A concentrated control force F_i generates a discontinuity in the third derivative. That is,

$$\frac{d^3 w}{dx^3}\bigg|_{x_i^+} = \frac{d^3 w}{dx^3}\bigg|_{x_i^-} - \frac{F_i}{EI}$$

We again choose the boundary conditions to be clamped at $x=0$ which results in a displacement w_{Fi0} of the form

$$W_{Fi0} = \frac{1}{EI}\left(\frac{x^3}{6} - \frac{x^2 x_i}{2}\right) \qquad \text{for } x \leq x_i$$

$$= \frac{x_i^2}{6EI}(x_i - 3x) \qquad \text{for } x > x_i \tag{31}$$

Force controls are most effective when applied at the edges. Thus, the control points are placed at the locations

$$x_i = (i-1)L/(n-1) \qquad i = 1,2,\ldots,n$$

Due to the need to maintain equilibrium, the minimum number of force control points is three. Table 1 shows that temperature controls can be as effective as force controls for reducing distortions. The disturbance along with the corrected shape resulting from seven thermal or force controls is shown in Fig. 1.

Feasibility of Corrective Temperatures

An important issue is the magnitude of the applied temperatures required to achieve the reductions shown in Table 1. Table 2 shows the temperatures for the 7-point case. To estimate the magnitude of the required inputs we make the following simplifying assumptions: All of the thermal control points are at $z_i = z_0$ and control the full cross-sectional area of the bottom flange so that $\Delta S_i = S_0$. For this case,

$$\gamma_i = \alpha_c E_c \Delta L_i / 2 z_0 E$$

The maximum temperature input from Table 2 is

$$\frac{\Delta T_3}{\Delta T_0} = 0.133\frac{\gamma_0}{\gamma_3} = 0.133\frac{\alpha_0}{\alpha_c}\frac{L}{\Delta L_3}\frac{E}{E_c}$$

If we assume that $E = E_c$ and require the maximum applied temperature not to exceed the disturbance temperature then

$$\Delta L_3 / L = 0.133(\alpha_0/\alpha_c)$$

If the control element is made of aluminum and the structure of low expansion composite material than α_0/α_c would be on the order of 0.01 and then $\Delta L_3/L$ is on the order of 0.001. This indicates that in fact it is feasible to control the beam by use of reasonably attained temperatures applied over relatively small control areas.

Equations for Shape Control of Structure—Discrete Formulation

Control by Applied Heating

In this section consider orbiting flexible structures modeled by finite elements. Following the development in the continuous formulation, it is assumed that both the distortion and the correction to the distortion are due entirely to flexible body effects so that they are orthogonal to all of the rigid-body modes (no translation or rotation of the c.g.).

$$\psi^T M R = 0 \tag{32}$$

and

$$U^T M R = 0 \tag{33}$$

where ψ is the distortion vector, M the mass matrix, R the matrix of rigid-body modes, and U the shape vector used to correct the distortion.

The rigid-body modes are assumed to be normalized such that

$$R^T M R = I \tag{34}$$

The displacement vector U is expressed as sum of the vector U_0 obtained by applying some convenient boundary conditions sufficient to suppress rigid-body motion plus a linear combination of the rigid-body modes

$$U = U_0 + \sum_{i=1}^{6} \alpha_i R_i \qquad i = 1,2,...,6 \tag{35}$$

or

$$U = U_0 + R\alpha \tag{36}$$

where $\alpha^T = [\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 \alpha_6]$.

The elements of α are evaluated using Eqs. (33) and (36). The result is

$$\alpha^T = -U_0^T M R \tag{37}$$

Similarly, for the distortion vector,

$$\psi = \psi_0 + \sum_{i=1}^{6} \beta_i R_i \tag{38}$$

$$\psi = \psi_0 + R\beta \tag{39}$$

so that

$$\beta^T = -\psi_0^T M R \tag{40}$$

where $\beta^T = [\beta_1 \beta_2 \beta_3 \beta_4 \beta_5 \beta_6]$.

The structure is controlled by applied temperatures at n control points. Denote the total (corrected) deformation vector of the structure by U_T. Then

$$U_T = \psi + \sum_{i=1}^{n} U_i \Delta T_i \tag{41}$$

where U_i is the displacement vector corresponding to a unit ΔT_i. The vectors U_i are obtained from Eqs. (35-37).

$$U_i = U_{0i} + \sum_{k=1}^{6} \alpha_{ik} R_k \tag{42}$$

or

$$U_i = U_{0i} + R\alpha_i \tag{43}$$

where U_{0i} is the displacement vector corresponding to a unit ΔT_i obtained by applying boundary conditions sufficient to suppress all rigid-body motion, and, from Eq. (37),

$$\alpha_i^T = -U_0^T M R \tag{44}$$

Table 2 Nondimensional control temperatures for thermal distortion control of beam

Control point No. i	Nondimensional temperature $\gamma_i \Delta T / \gamma_0 \Delta T_0$[a]
1	0.125
2	0.123
3	0.133
4	0.063
5	−0.008
6	0.002
7	−0.001

[a] $\gamma_i = \alpha_c E_c \Delta L_i / 2z_0 E_0$; $\gamma_0 = \alpha_0 L / 2z_0$; c = control element material; and 0 = structural material.

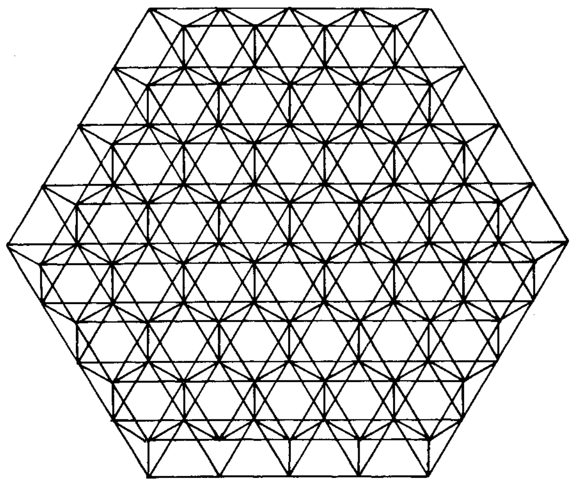


Fig. 2 Tetrahedral truss antenna reflector.

The control problem is to determine those temperatures which minimize

$$U_{\rm rms}^2 = U_T^T B U_T \tag{45}$$

where U_T is defined in Eq. (41) and B is any symmetric positive definite matrix. Substituting Eq. (41) into (45) and minimizing $U_{\rm rms}^2$ with respect to each ΔT_i gives

$$\left(\psi^T + \sum_{j=1}^{n} U_j^T \Delta T_j\right) B U_i = 0 \qquad i = 1,2,...,n \tag{46}$$

This equation may be written as

$$A \Delta T = r \tag{47}$$

where the element in the ith row and jth column of A is

$$a_{ij} = U_i^T B U_j \tag{48}$$

and

$$r_i = -U_i^T B \psi \tag{49}$$

$$\Delta T^T = [\Delta T_1 \Delta T_2 ... \Delta T_n] \tag{50}$$

Control by Applied Forces

In this formulation the disturbance is corrected by the application of n static control forces. The corrected deformation is given by

$$U_T = \psi + \sum_{i=1}^{n} U_i F_i \tag{51}$$

where F_i is the force at the ith control point and U_i the displacement vector corresponding to a unit value of F_i.

It is convenient to define a vector of control forces having the same number of elements as the displacement vector.

$$F=\sum_{i=1}^{n} F_i L_i \tag{52}$$

where L_i is a location vector which has zeros everywhere except for the value of 1.0 in the position corresponding to the location of the ith control force.

The requirement that the control forces be in equilibrium is expressed by

$$R_k^T F=0 \qquad k=1,2,\ldots,6 \tag{53}$$

The control forces are required to minimize an objective function Γ which contains the distortion measure augmented by the force equilibrium constraints. That is

$$\frac{\partial \Gamma}{\partial F_i}=0, \quad \frac{\partial \Gamma}{\partial \lambda_k}=0 \qquad i=1,2,\ldots,n \qquad k=1,2,\ldots,6 \tag{54}$$

where

$$\Gamma=U_T^T B U_T-\sum_{k=1}^{6} \lambda_k R_k^T F \tag{55}$$

and λ_k is a Lagrange multiplier.

Substituting Eqs. (51), (52), and (55) into Eq. (54) we obtain

$$\psi^T B U_j+\sum_{i=1}^{n} U_i^T B U_j F_i-\frac{1}{2}\sum_{k=1}^{6} R_k^T L_j \lambda_k=0 \qquad j=1,2,\ldots,n \tag{56}$$

and

$$R_m^T \sum_{i=1}^{n} L_i F_i=0 \qquad m=1,2,\ldots,6 \tag{57}$$

Multiplying Eq. (57) by −½ permits Eqs. (56) and (57) to be written in the following compact form:

$$\begin{bmatrix} A & C \\ C^T & 0 \end{bmatrix} \begin{Bmatrix} F \\ \lambda \end{Bmatrix} = \begin{Bmatrix} r \\ 0 \end{Bmatrix} \tag{58}$$

where A is defined by Eq. (48), and

$$C_{ij}=-\tfrac{1}{2} L_i^T R_j \tag{59}$$

$$F^T=[F_1 F_2 \ldots F_n] \tag{60}$$

$$\lambda^T=[\lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5 \lambda_6] \tag{61}$$

Application to Radiometer Antenna Reflector

The procedures described in the preceding section have been implemented in the EAL structural analysis computer program.[15] Listings of the EAL input (runstreams) for applying the procedure to a radiometer antenna reflector using control by applied heating and by applied forces are available in Ref. 16.

Model Description and Loading

The discrete formulation is applied to a finite element model of the reflector of a 55-m radiometer antenna shown in Fig. 2. The reflector is built up from tetrahedral truss modules and the model consists of 109 grid points and 420 rod elements. The structure is composed of graphite-epoxy and the control elements are aluminum. The coefficients of thermal expansion of graphite-epoxy and aluminum is 0.23 and $23.0\times10^{-6}/°C$, respectively. Young's modulus is 73. 0 GPa for both materials. The antenna is in low-Earth orbit at an altitude of 216 miles and is in an Earth-facing orientation. The thermal loading consists of a combination of solar, Earth-reflected (albedo), and Earth-emitted heat flux. A transient thermal analysis of the reflector for a complete orbit was performed using the SPAR finite element thermal analysis program[17] using time-dependent heat flux data supplied by Ref. 18. The thermal analysis output was examined and a worst-case temperature distribution (the one with the largest gradients) was selected for the calculation of thermal distortion. The temperature history and location of the design point are shown in Fig. 3. Only the z component of the displacement was considered to need correction so that the B matrix of Eq. (45) is a diagonal matrix where every third term on the diagonal is unity and the rest are zero.

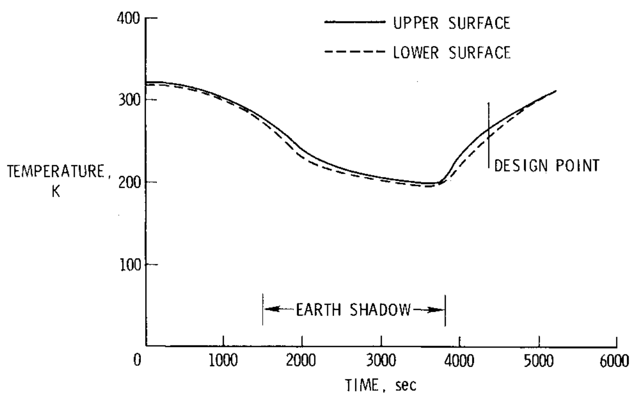
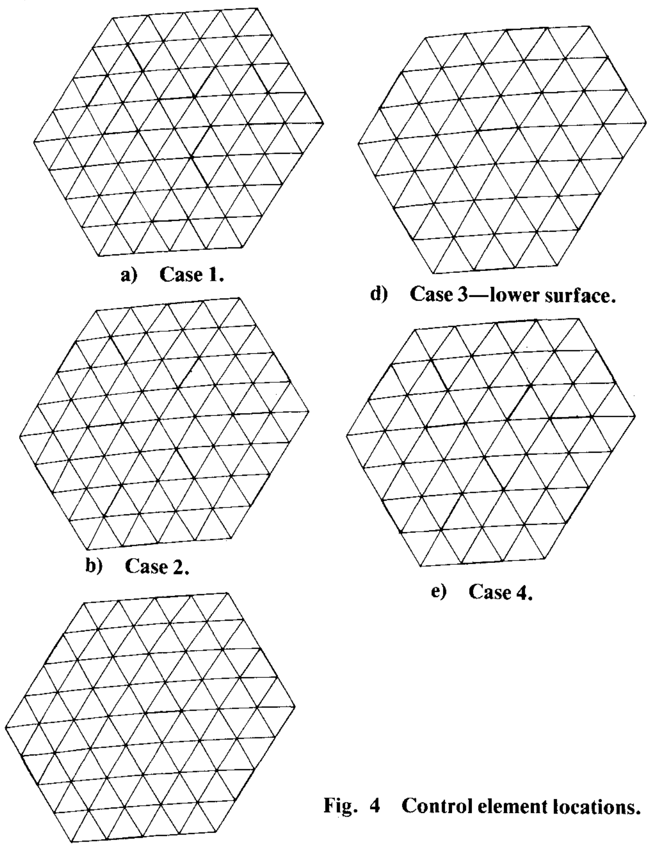


Fig. 3 Temperature history for antenna reflector.



a) Case 1.

b) Case 2.

c) Case 3—upper surface.

d) Case 3—lower surface.

e) Case 4.

Fig. 4 Control element locations.

Control Point Locations

Four sets of control point locations were considered. These are designated cases 1-4 and illustrated in Fig. 4. In case 1, 12 control points are clustered near the center of the reflector with a few near the outer portion—all in the upper surface. In case 2, 12 control points are nearly symmetrically arranged around the outer rim and on the diagonals about midway between the center and outer rim—again all on the upper surface. In case 3 there are 14 control points (seven on each surface) with points located around the outer rim and near the center. In case 4 there are 12 control points on the lower surface, arranged in essentially the same position as in case 2.

Table 3 Influence of control point locations on effectiveness of distortion control by applied temperatures

Case No.	Description	Range of ΔT's[a]	U_{rms}[b]	G[c]
—	Uncorrected	—	0.0142	—
1	Control points at interior upper surface	−0.11 to +0.11	0.00625	0.440
2	Control points at outer ring in upper surface	−0.56 to +1.7	0.00488	0.344
3	Control points at outer ring in upper and lower surfaces	−0.72 to +2.5	0.00453	0.319
4	Control points at outer ring in lower surface	−0.11 to 0.056	0.00585	0.412

[a] $\Delta T = T - 27°C$. [b] $U_{rms} = \left[\sum_i U_{zi}^2\right]^{1/2}$ (cm). [c] $G = \dfrac{U_{rms\ corrected}}{U_{rms\ uncorrected}}$.

Results and Discussion

Results of the study are summarized in Table 3. The first two columns contain descriptions of the four location cases. The third column indicates the range of applied temperatures required to minimize thermal distortion. In all cases some negative temperature differences are required, however, these are relative to the reference temperature of 27°C and do not represent particularly low temperatures. The fourth column contains the corrected distortion expressed in terms of the rms measure for normal components of deflections. The fifth column contains the values of G—the ratio of the corrected-to-initial rms distortion. From Table 3 it is evident that cases 2 and 3 had the most effective distribution of control points while case 1 was least effective. Partial depictions of distorted and corrected shapes for the four cases are shown in Fig. 5. For simplicity these illustrations contain cross sections of the shapes corresponding to a section through a diagonal of the reflector (along the line $y=0$). These figures are intended to pictorially convey the nature of shape control achievable by corrective heating. Since the figures do not include all the points in the structure, they are not as suitable as the values of G in Table 3 for comparing the effectiveness of the control point distributions.

The force control procedure was applied to the reflector dish with 12 control forces applied in the z direction in the upper surface as shown in Fig. 6. The result was a value of G of 0.499 compared to 0.344 for the corresponding thermal control case. Sections of the uncorrected and corrected shapes are shown in Fig. 6. This result indicates that force control is somewhat less effective than thermal control for the same

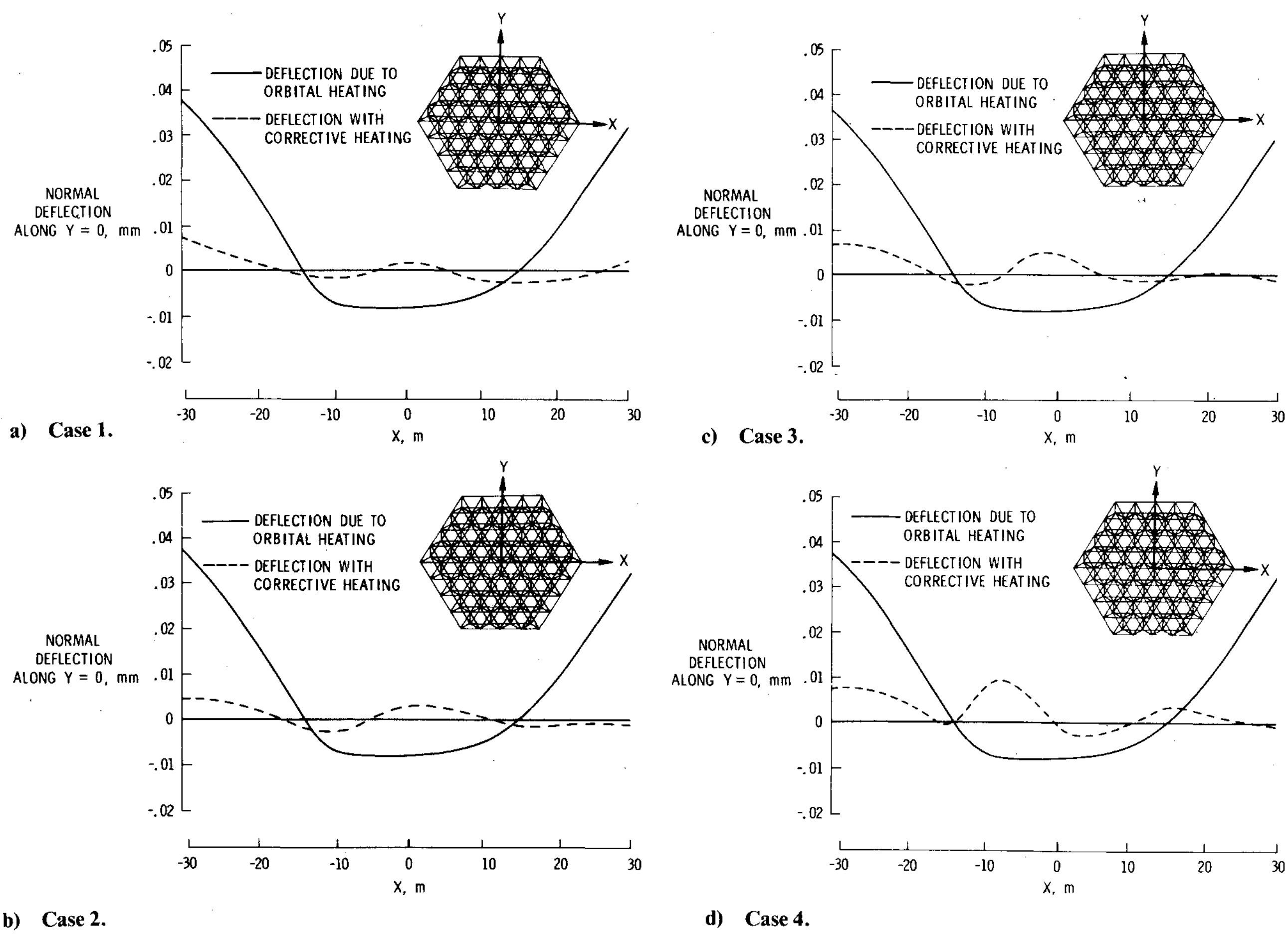


Fig. 5 Control of antenna surface distortion by applied temperatures.

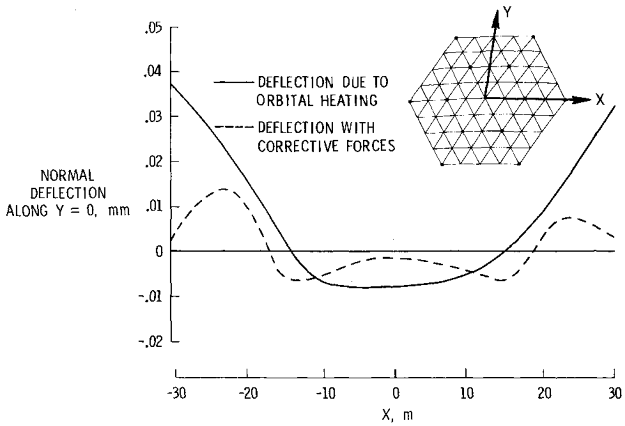


Fig. 6 Control of antenna surface distortion by applied forces.

number of control points—a fact which is mostly due to the need for the control forces to be in equilibrium so that 6 of the 12 forces are essentially unavailable for controlling distortion.

Concluding Remarks

This paper describes an analytical procedure for the static shape control of quasisteady distortions in large flexible space structures with applications to large space antennas. The procedure is based on prescribing temperatures in control elements which have much higher coefficients of thermal expansion than the main structure. The temperatures at the control elements are determined to minimize the overall distortion of the structure from its ideal shape. A matrix equation is derived which is easily solved for the set of optimum control temperatures.

The paper contains two formulations of the procedure: one for continuous structures by differential equations, and the other for discrete (finite element modeled) structures governed by matrix equations. For comparison purposes, analogous equations have been developed for static shape control by applied forces. The equations from the continuous formulation have been applied to shape control of a flexible beam distorted by nonuniform heating. The discrete formulation has been implemented in a general-purpose finite element structural analysis computer program and demonstrated for shape control of a 55-m radiometer antenna reflector subjected to orbital heating.

Results of the beam example demonstrated that shape control by applied temperatures was an effective approach. Use of only seven control elements was able to achieve a factor of 50 reduction in thermal distortion. A study was carried out for the antenna to assess the effect of control point distribution on shape control effectiveness. Results were obtained for four different sets at locations of controls. For the best location set, a factor of three reduction in the distortion was achieved with 14 control points.

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